Example E.3 (ADDRESS MODEL) Address models link attributes to preference without imposing the restriction that some attributes strictly dominate others as in the lexicographic model. Suppose we have n alternatives and each alternative has m attributes that take on real values. Alternatives can then be represented as n points, $z_1, \ldots, z_n$, in $\Re^m$, which is called *attribute space.* For example, in a travel context attributes may include departure time, arrival time, and price.

Each customer has an ideal point ("address") $y \in \Re^m$, reflecting his most preferred combination of attributes (such as an ideal departure time, arrival time, and price). A customer is then assumed to prefer the product closest to his ideal point in attribute space, where distance is defined by a metric ρ on $\Re^m \times \Re^m$ (such as Euclidean distance). These distances define a preference relation, in which $z_i \succ z_j$ if and only if $\rho(z_i, y) < \rho(z_j, y)$; that is, if z_i is "closer" to the ideal point y of the customer.

Utility Functions

Preference relations are intimately related to the existence of utility functions. Indeed, we have the following theorem (See Kreps [313] for a proof.):

THEOREM E.3 *If X is a finite set, a binary relation $\succ$ is a preference relation if and only if there exists a function $u : X \to \Re$ (called a **utility function**), such that*

$$x \succ y \quad \textit{iff} \quad u(x) > u(y).$$

Intuitively, this theorem follows because if a consumer has a preference relation, then all products can be ranked (totally ordered) by his preferences; a utility function then simply assigns a numerical value corresponding to this ranking. Intuitively, one can think of utility as a measure of "value," though in a strict sense its numerical value need not correspond to any such tangible measure. Theorem E.3 applies to continuous sets X (such as travel times or continuous amounts of money) as well under mild regularity conditions, in which case the utility function $u(\cdot)$ is then continuous. The following examples illustrate the construct of utility:

Example E.4 A utility function corresponding to the lexicographic model of Example E.2 can be constructed as follows: Suppose there are n alternatives with m attributes each. Let the attributes be ordered so that 1 represents the highest-valued attribute and m the lowest. Let $a_k(x), k = 1, \ldots, m$, be binary digits representing whether alternative x, possesses attribute k. Then a utility satisfying Theorem E.3 is the binary number,

$$u(x) = a_{x1} a_{x2} \cdots a_{xm}.$$

Maximizing over these utilities leads to the same customer decisions as the lexicographic model.

Example E.5 Consider the address model of Example E.3. Again, Theorem E.3 guarantees that an equivalent utility maximization model exits that generates the same choices. In this case, it is easy to see that for customer y the continuous utilities

$$u(z) = c - \rho(z, y),$$

where c is an arbitrary constant, produce the same decision rule as the address model.